Letter to the Editor

An unusual aberration of very large liquid mirror telescopes

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Abstract. Large Liquid Mirror telescopes, and any other telescope rotating around the line of sight, like altasimuth ones, are affected by a small blurring due to the light aberration when the light is reflected by the moving edges of the mirror. For ground-based liquid mirrors such effect dominates over the diffraction size for diameter of the order of some tens of meters, at optical wavelength and for reasonable choices of the focal length. The same effect, for an altasimuth telescope, appears negligible. Also a fundamental limitation on the spectral resolution attainable by a spectrograph coupled with such a telescope is pointed out, but it appears negligible by several order of magnitudes.

Key words: liquid mirrors – telescopes

When the source or the observer of a light phenomenon is moving, classic laws of reflection are inadequate and should be replaced by relativistic optics relationships. A pencil beam reflected by a moving mirror in a direction perpendicular to its normal with speed v is displaced with respect to the classical prediction of an angular amount \( \theta \) given (in the approximation \( v \ll c \)) by:

\[
\theta = \frac{2v}{c}
\]

and, moreover, it is wavelength shifted by the transverse Doppler effect, of an amount given by:

\[
\frac{\Delta \lambda}{\lambda} = \frac{2v^2}{c^2}
\]

(see for example Ditchburn, 1953).

The same effect will affect the rays coming from the edge of a D diameter telescope rotating at an angular speed \( \omega \) leading to an apparent blurring of the image given by the following relationship:

\[
\theta = \frac{\omega D}{c}
\]

It is worthwhile to point out that such aberration cannot be compensated by re-phasing of the incoming beam. It is easy to show, in fact, that the displacement \( \theta \) corresponds to a wavefront gradient \( \nabla W \) such that \( W \) is not continuous (see also Fig.1).

Fig. 1. The discontinuous wavefront generated by integration of the vectorial field is shown together with a sketch of the displacements of the rays reflected by the rotating mirror.

For a ground-based liquid mirror, parabolically shaped via centrifugal forces, a relationship between the focal length \( f \) of the mirror itself and its angular velocity is given by (Borra, 1982):

\[
f = \frac{g}{2\omega^2}
\]

where \( g \approx 9.81 \text{ m} \cdot \text{s}^{-2} \) is the Earth acceleration gravity. Coupling eqs. (2) and (3) one obtain the minimum focal length \( f_{\text{min}} \) required to avoid that this sort of aberration becomes comparable to the diffraction limit \( \lambda/D \), obtaining:

\[
f_{\text{min}} = \frac{g}{2\lambda^2c^2}D^4
\]

In Fig. 2 \( f_{\text{min}} \) is plotted against \( D \) ranging from 1 to 100 meters assuming \( \lambda = 500 \text{nm} \). As one can see that diameters
greater than a few tens of m lead to primary mirror focal length of various km. In Fig. 3, in fact, the focal ratio $F/\lambda$ of the primary mirror is plotted against $D$. Realistic values are confined to the same limits.

![Graph](image1)

**Fig. 2.** The minimum focal length of a rotating liquid mirror required to avoid that the aberration described in the paper dominates over the diffraction limit for $\lambda = 500$ nm.

![Graph](image2)

**Fig. 3.** The minimum $F/\lambda$ for the same conditions explained in Fig. 1.

Lunar-based liquid mirror telescopes (Borra, 1991) or large orbiting liquid mirror telescopes (Borra, 1992) are interested by a $g$ value much lower (from six times lower for the lunar case, to several order of magnitude lower for the orbiting case). Being the aberration proportional to the $g$ value the effect becomes significantly smaller.

The transverse Doppler effect broadens the spectrum of the collected radiation by an absolutely negligible amount even at extremely large diameter; coupling eqs. (2) with (4) and assuming an $F/\lambda \approx 1$ a broadening of the order of:

$$\frac{\Delta \lambda}{\lambda} \approx 2.5 \cdot 10^{-11} D$$  \hspace{1cm} (6)

for a liquid mirror telescope can be evaluated. Much smaller values are expected for an altazimuth telescope.

References

Ditchburn R.W (1953) "Light" Blackie and Son, Ltd. Bishopbriggs, Glasgow, Scotland, chapter XI, see especially eqs.11(31), 11(32) and examples 11(ix), 11(x)

Very large liquid mirrors, judged as feasible (Borra, Beauchemin & Lalande, 1985) are affected by a small blurring due to the mirror rotation. This effect becomes important for diameters of the order of some tens of meter. At this diameter range the quality of the image is seeing-dominated, but speckle and adaptive-optics techniques, should take into account this effect. It is noticed that this blurring, even if it should look like conventional defocussing, is of a substantially different nature with respect to aberrations introduced by the departure of the mirror surface from the theoretical figure (like the one introduced by Coriolis forces or ripples on the liquid surface)

Also altazimuth large telescopes are affected by such an effect. Nevertheless it is easy to show that this is negligible for even very large diameter, tracking as close to the zenith as only few arcmin.